Rate Transient Analysis

A Petroleum Engineering Reference Guide

whitson

Flow Regimes

Infinite acting flow, often referred to as transient flow, is the flow regime that ends as the pressure transient reaches *one* reservoir boundary.

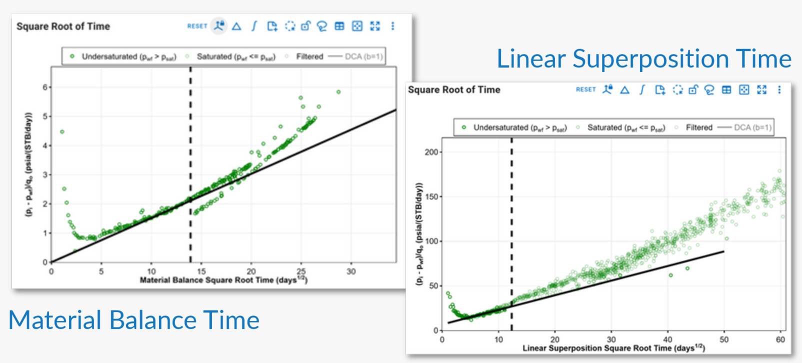
Transitional flow is the flow regime that starts as the pressure transient reaches *one* reservoir boundary and ends when the pressure propagation reaches *all* reservoir boundaries.

Boundary dominated flow, also called pseudo-steady state, is the flow regime that starts when the wellbore pressure response is affected by *all* reservoir boundaries.

Superposition

Superposition accounts for changing rates and pressures. Common time functions include material balance time and linear superposition time.

Caution: The choice of superposition time function can bias interpretation—square-root-time often makes data appear as linear flow, log-time as radial flow, and material-balance time as boundary-dominated flow.



Linear Superposition Time

Material Balance Time

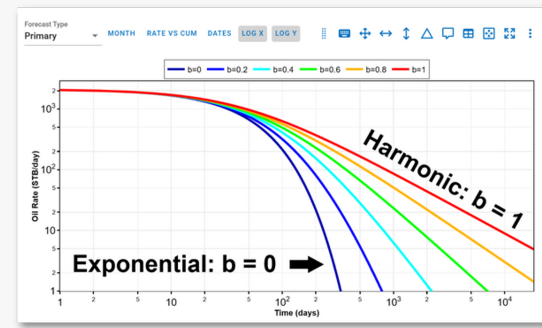
Decline Curve Analysis (DCA)

Arps Equation

$$q(t) = q_i(1 + bD_it)^{-\frac{1}{b}}$$

q_i = initial flow rate
a_i = nominal decline rate at time zero
b = rate exponent
$q(t)$ = flow rate at time t

Type	Decline Rate	Producing Rate, q	Elapsed Time, t	Cum. Production, Q_t
Exponential b = 0	$a_t = ln(\frac{q_i}{q_t})/t$	$q_i e^{-a_i t}$	$ln(\frac{q_i}{q_t})/a_i$	$\frac{q_i - q_t}{a_i}$
Hyperbolic b > 1	$\frac{a_i}{a_t} = (\frac{q_i}{q_t})^b$	$q_i(1 + ba_it)^{-1/b}$	$\frac{(q_i/q_t)^b - 1}{ba_i}$	$\frac{q_i}{a_i(1-b)}(1 - \frac{q_t}{q_i}^{1-b})$
Harmonic b = 1	$\frac{a_i}{a_t} = \frac{q_i}{q_t}$	$q_i(1 + a_it)^{-1}$	$\frac{(q_i - q_t)}{a_i q_t}$	$\frac{q_i}{D_i} ln(\frac{q_i}{q_t})$



b-factor Rules of Thumb	
b = 0	single-phase oil, or gas at high pressure
b = 0.1-0.4	solution gas drive
b = 0.4-0.5	single-phase gas flow
b = 0.5-1	layered reservoirs
b >= 1	infinite acting, or transitional flow

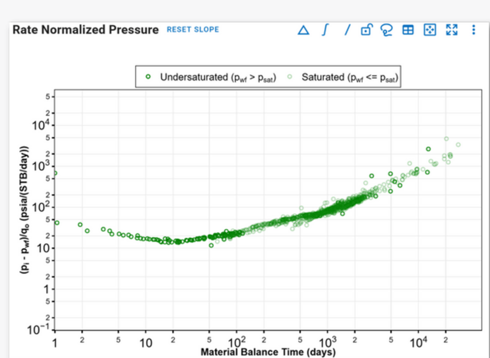
Rate Normalized Pressure (RNP)

$$RNP = \frac{\Delta p}{q} = \frac{p_i - p_{wf}}{q}$$

Rate Normalized Pressure (RNP) is very useful for production analysis where flowing pressures and rates change through time.

It is defined as the flowing pressure drop divided by rate.

RNP is the inverse of PNR.

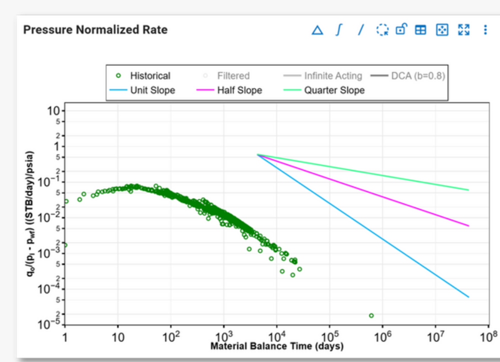


Pressure Normalized Rate (PNR)

$$PNR = \frac{q}{\Delta p} = \frac{q}{p_i - p_{wf}}$$

Pressure normalized rate is very useful for production analysis where flowing pressures and rates change through time. It is defined as the rate divided by flowing pressure drop.

PNR is the inverse of RNP.

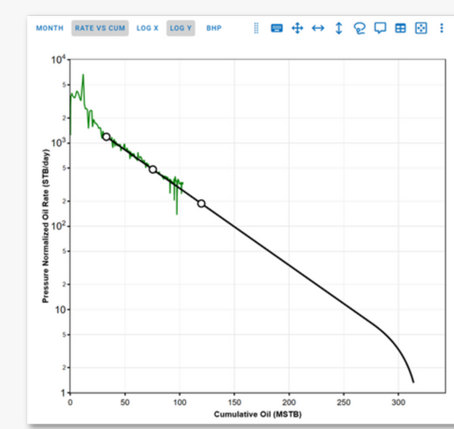


PNR DCA

$$q_{PNR} = \frac{q}{p_i - p_{wf}} \times (p_i - p_{wf,min})$$

Xie (2023) identified that DCA in unconventional wells can overestimate EUR due to extended early-time flat production. Using the PNR flowrate—calculated by multiplying PNR with a constant pressure drawdown—offers a more consistent alternative.

Where q_{PNR}, represents the PNR flowrate at a constant pressure drawdown and $p_{wf,min}$ is the estimated abandonment BHP.



Pseudo-Pressure

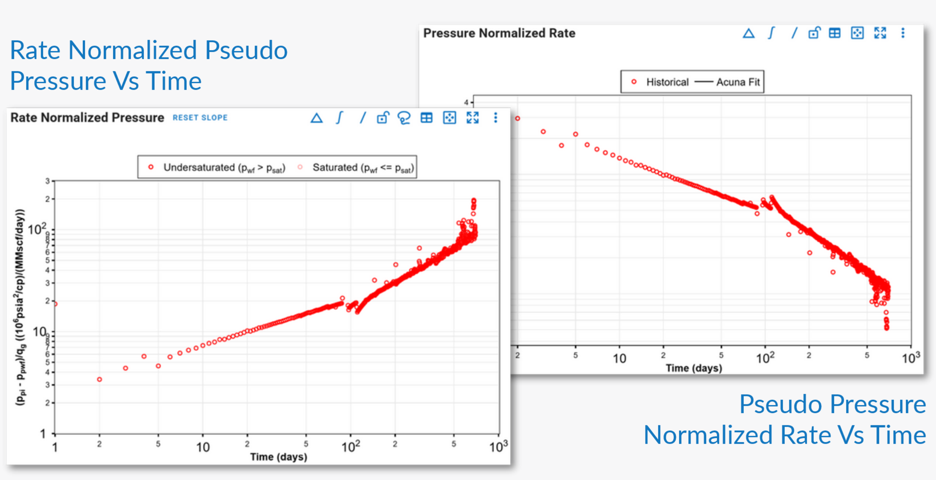
Compared to liquids, gas compressibility, density, and viscosity vary significantly with pressure, which is why pseudo-pressure is introduced to linearize gas flow equations and express them in a form similar to those for incompressible fluids.

Real gas pseudo-pressure drop defined as

$$\Delta p_p = p_{p_i} - p_{p_{wf}} = 2\int_{p_{wf}}^{p_i} \frac{p}{\mu_g Z} dp$$

To account for pressure-dependent permeability, this is defined as:

$$p_p = 2\int_0^p \frac{pk_m(p)}{\mu_g Z} dp$$



Rate Normalized Pseudo Pressure Vs Time

Pseudo Pressure Normalized Rate Vs Time

Flowing Material Balance (FMB)

Flowing material balance is a diagnostic and analytical method used to estimate the contacted pore volume (V_p)—essentially the "size of the tank", using only production and pressure data.

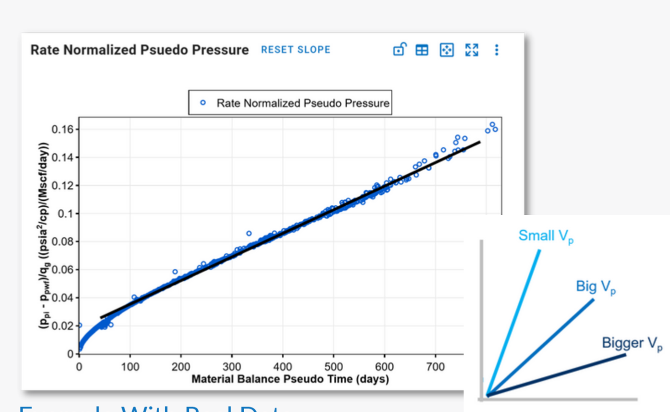
Assumes that the **reservoir is in boundary dominated flow,** meaning pressure response is influenced by reservoir boundaries rather than transient behavior. Applying the method during infinite-acting flow often lead to conservative estimates.

A key difference between FMB and RTA is that RTA estimates both the linear flow parameter (LFP) and the contacted pore volume (V_p), while FMB resolves only the contacted pore volume.

Gas FMB

$$\frac{\Delta m_{wf}(t)}{q_g(t)} = m_{pss}t_{ma} + b_{pss}$$

The work presented by Palacio and Blasingame (1993) involves plotting rate-normalized pseudopressure vs. material balance pseudotime (t_{ma}). During pseudosteady state flow, this results in a straight line with slope inversely proportional to the contacted pore volume. The method is iterative, so the points on the diagnostic plot adjust as the interpretation is refined.

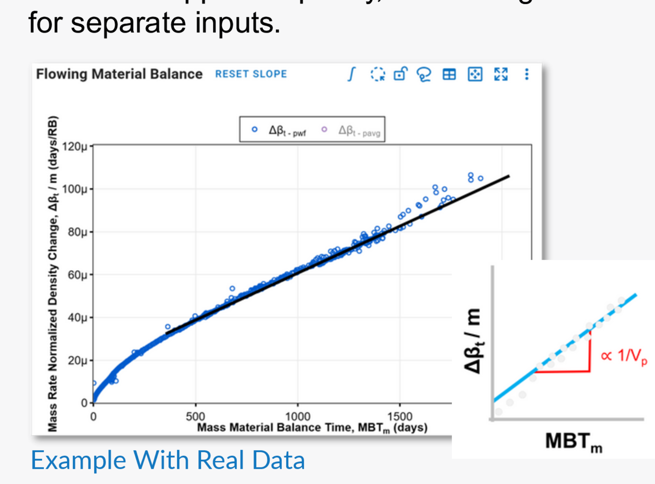


Example With Real Data

Multiphase FMB

$$\frac{\beta_i - \beta_{well}}{\dot{m}(t)} = \frac{1}{V_p}MBT_M + \frac{1}{b_M}$$

The multiphase FMB method proposed by Thompson & Ruddick (2022) estimates contacted pore volume without needing relative permeabilities. All transport-related terms (permeability, relative permeability, and pressure-dependent permeability) are built into the rates and never appear explicitly, eliminating the need for separate inputs.



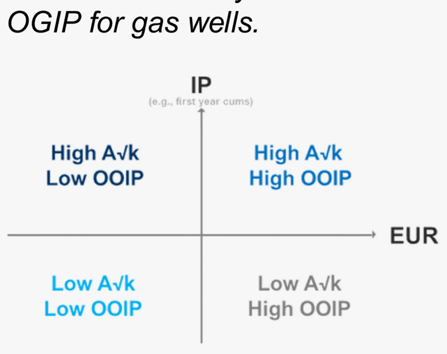
Example With Real Data

RTA Basics

RTA can be used to quantify the linear flow parameter (LFP, also known as A√k) and the contacted pore volume (V_p).

Linear Flow Parameter	**LFP (or A√k)**	Quantified during **infinite acting**, linear flow
Contacted Pore Volumes	**OOIP (or OGIP)**	Quantified during **boundary dominated** flow

** Contacted OOIP commonly used for oil wells, contacted OGIP for gas wells.*



What is it used for?

- Well performance comparison
- Forecasting
- Completion effectiveness & frac optimization
- Production optimization/drawdown management
- Calibration/starting point for advan. simulation studies
- Identifying well interference and depletion effects
- Reserves booking support

Classical RTA

$$\frac{p_i - p_{wf}}{q_o} = m_{CR}\sqrt{t} + b'$$

This is the constant rate solution. The constant pressure solution is π/2 larger!

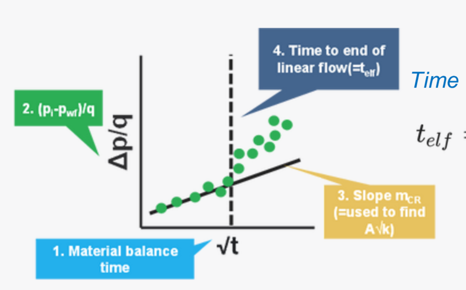
For oil:

$$m_{CR} = \frac{19.927B_o}{h_f x_f n_f \sqrt{k}}\sqrt{\frac{\mu_{oi}}{\phi c_{ti}}}$$

For gas*:

$$m_{CR} = \frac{200.8T_R}{h_f x_f n_f \sqrt{k}} \frac{1}{\sqrt{(\phi \mu_g c_t)_i}}$$

*For gas pseudo-pressure is used



Time to end of linear flow:

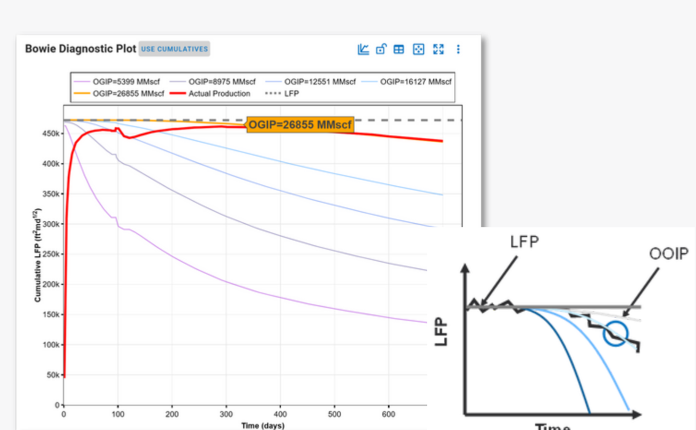
$$t_{elf} = \frac{(\phi \mu c_t)_i}{k}\left(\frac{L_w}{0.226 * n_f}\right)^2$$

Numerical RTA

$$LFP = 4n_f x_f h\sqrt{k}$$

Correcting for changing rate, pressure and multiphase flow effects is not possible — doing so requires knowing saturations and PVT properties across space and time. With complex PVT, relative permeability, and hysteresis effects, analytical solutions break down (Jones, 1985). The only viable approach is a full numerical solution using actual bottomhole pressure data.

To solve this, Bowie and Ewert (2020) introduced Numerical RTA—a systematic method using a numerical model to resolve a linear flow parameter (LFP) and contacted pore volume. The corresponding diagnostic plot is shown below.

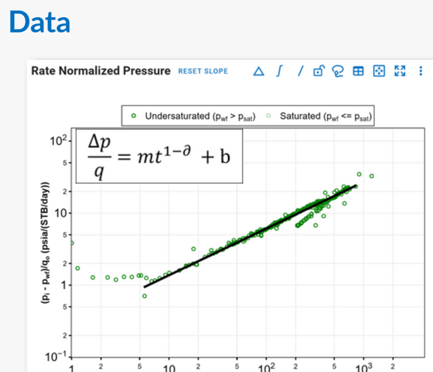


Fractional RTA

While **Classical RTA** solves for equally spaced fracture networks and resolves LFP = A√k = (δ = 0.5).
Fractional RTA (Acuna, 2020) solves for complex system of fractures and resolves LFP = Ak^{δ} (any value of δ).

Estimate of δ from Prod Data

An estimate of δ can be obtained by plotting rate normalized pressure (RNP) versus time followed by a best fit of the power-law function as shown in the plot to the right



δ and Arps' b-factor

There is a relationship between the δ-parameter and Arps' b-factor.

$$\delta = 1 - \frac{1}{b} \rightarrow b = \frac{1}{1 - \delta}$$

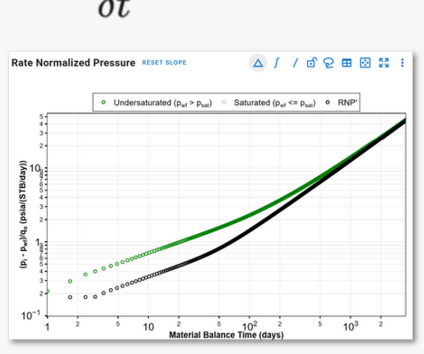
Remember that if you use the δ-parameter parameter to constrain b-factors used in DCA, you must resolve the δ-parameter in real time (not material balance, since b can not be below 1 in material balance time).

RTA Derivatives

The logarithmic derivative used in RTA applied directly on the rate normalized pressure (RNP) data is given

$$RNP' = \frac{\delta RNP}{\delta log(t)} = t\frac{\delta RNP}{\delta t}$$

Derivatives highlight flow regime transitions by amplifying changes in the data (but also amplify noise), making them useful for identifying flow regimes

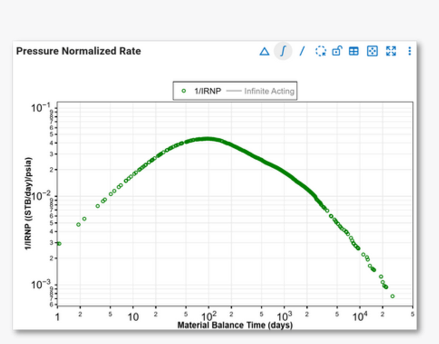


RTA Integrals

The integral method in RTA is applied using the pressure integral

$$IRNP = \frac{1}{t_{MB}}\int_0^{t_{MB}} \frac{p_i - p_{wf}(\tau_{MB})}{q(\tau_{MB})} d(\tau_{MB})$$

Integrals smooth out short-term fluctuations and reduce noise, offering a clearer view of the overall reservoir response.



Bonus Equations

Darcy's Law

$$\vec{v_o} = -\frac{kk_{ro}}{\mu_o}\nabla(p_o + \rho_o gh)$$

$$\vec{v_g} = -\frac{kk_{rg}}{\mu_g}\nabla(p_g + \rho_g gh)$$

$$\vec{v_w} = -\frac{kk_{rw}}{\mu_w}\nabla(p_w + \rho_w gh)$$

Producing GOR

$$R_p = \frac{q_{\bar{g}g} + q_{\bar{g}o}}{q_{\bar{o}g} + q_{\bar{o}o}} = [1 + \alpha r_s]^{-1}[R_s + \alpha]$$

$$\alpha = \frac{k_{rg}}{k_{ro}} \cdot \frac{\mu_o B_o}{\mu_g B_{gd}}$$

For $r_s = 0$

$$R_p = \frac{q_{\bar{g}g} + q_{\bar{g}o}}{q_{\bar{o}o}} = R_s + \alpha$$